OCR
Oxford Cambridge and RSA

Wednesday 7 June 2017 – Morning

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

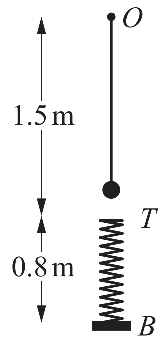
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle of mass 0.2 kg is moving with speed $4\,\mathrm{m\,s^{-1}}$ in a straight line on a smooth horizontal plane. A horizontal impulse of magnitude 1.2 N s acts on the particle. After the impulse acts the particle is moving with speed $5\,\mathrm{m\,s^{-1}}$.

Find the angle between the directions of motion of the particle before and after the impulse acts. Find also the angle between the direction in which the impulse acts and the initial direction of motion of the particle. **[5]**

2



A particle of mass m kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $24mg$ N. The other end of the string is attached to a fixed point O. Directly beneath O there is a light elastic spring, of natural length 0.8 m and modulus of elasticity $32mg$ N. The bottom of the spring is attached to a fixed point B and the top of the spring is at a point T, 1.5 m vertically below O; the spring is constrained to remain vertical. The particle is projected from O with speed $0.7\,\mathrm{m\,s^{-1}}$ vertically downwards. When the particle reaches T it becomes attached to the spring and it remains attached to the spring throughout the subsequent motion. The diagram shows the position as the particle first approaches T.

(i) Show that the speed of the particle at the instant when it becomes attached to the spring is $3.5\,\mathrm{m\,s^{-1}}$. **[3]**

(ii) Find the distances below O at which the particle is instantaneously at rest in the subsequent motion. **[5]**

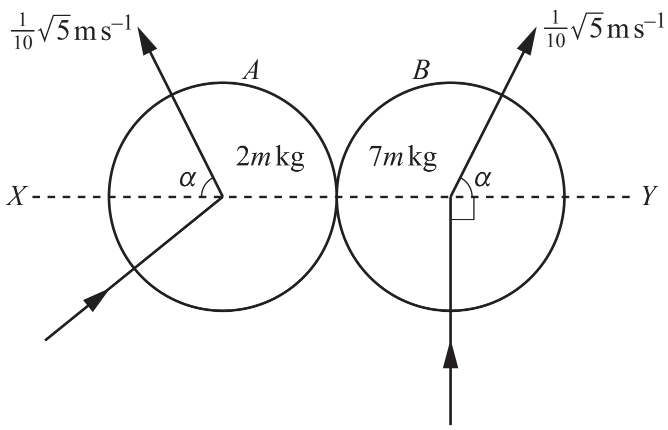
3 A particle of mass 0.2 kg travels in a straight line on a smooth horizontal surface. At time t seconds it is x m from a fixed point O and is moving away from O with velocity $v\,\mathrm{m\,s^{-1}}$. A force of magnitude $\frac{1}{2}\left(12-\frac{1}{4}v\right)^{\frac{1}{2}}$ N acts on the particle in the direction of motion. At time $t = 0$ the particle is at O and has velocity $12\,\mathrm{m\,s^{-1}}$.

(i) State the maximum possible velocity of the particle. **[1]**

(ii) Find an expression for v in terms of t, valid while the particle is accelerating. **[6]**

(iii) Hence find the distance travelled by the particle as its velocity increases from $12\,\mathrm{m\,s^{-1}}$ to $32\,\mathrm{m\,s^{-1}}$. **[4]**

4



Two uniform smooth spheres, A and B, of equal radius, with masses $2m$ kg and $7m$ kg respectively, collide on a horizontal surface. Immediately before the collision B is moving in a direction perpendicular to the line of centres, XY. Immediately after the collision both A and B are moving with speed $\frac{1}{10}\sqrt{5}\,\mathrm{m\,s^{-1}}$; they are moving on the same side of XY and are both moving in directions making an angle α with XY, where $\tan\alpha = 2$ (see diagram).

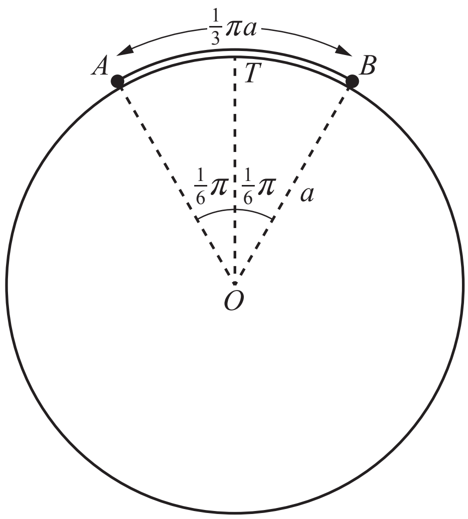
(i) Find the speed and direction of motion of A before the collision with B. Find also the coefficient of restitution in the collision. **[7]**

Subsequently another uniform smooth sphere, C, of mass $3m$ kg and with the same radius as A and B, collides with A. The line of centres of the collision between C and A is parallel to XY; immediately before this collision C is travelling with speed $U\,\mathrm{m\,s^{-1}}$ from left to right parallel to XY. This collision is perfectly elastic.

(ii) Explain why A will have a second collision with B if U is large enough. **[1]**

(iii) Find the greatest value of U for which A will **not** subsequently have a second collision with B. **[6]**

5



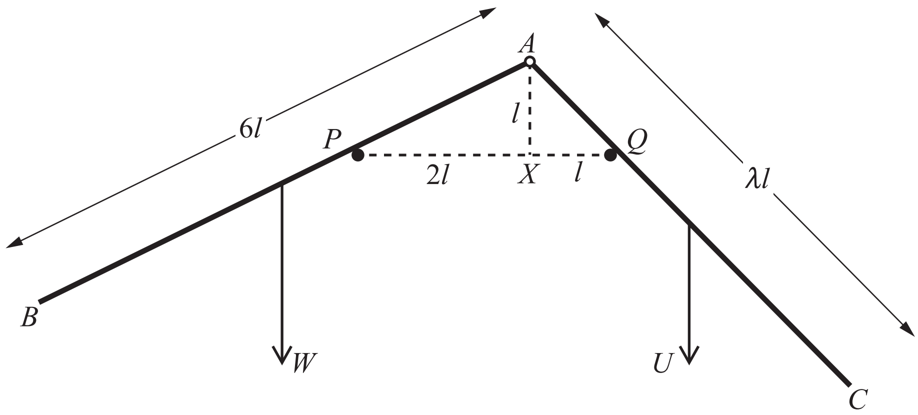
Particle A of mass $3m$ and particle B of mass $2m$ are joined by a light inextensible string of length $\frac{1}{3}\pi a$. The particles and the string rest in a vertical plane on the surface of a smooth cylinder, of radius a, which has its axis horizontal; O is the centre of the vertical cross-section of the cylinder containing the particles and T is the uppermost point on the surface of the cylinder. The string is taut and radii OA and OB each make an angle of $\frac{1}{6}\pi$ radians with OT (see diagram). A and B are released from rest, and after t seconds angle AOT is $\left(\frac{1}{6}\pi + \theta\right)$ radians and both A and B are moving with speed v.

(i) Show that, while A and B remain in contact with the cylinder,

$$v^2 = \frac{2}{5}ag\left(5\cos\frac{1}{6}\pi - 3\cos\left(\frac{1}{6}\pi + \theta\right) - 2\cos\left(\frac{1}{6}\pi - \theta\right)\right).$$ **[4]**

(ii) Find an expression in terms of m and g for the force exerted on A by the cylinder at the instant when B is passing through T. **[4]**

6

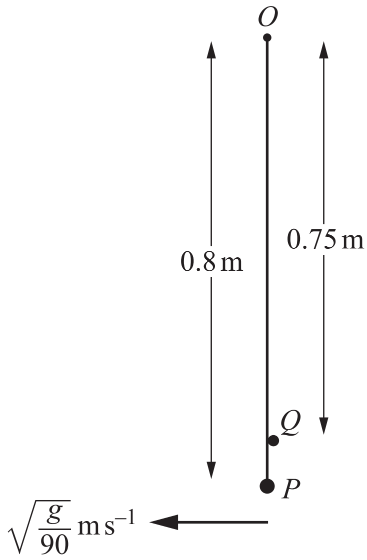


Two uniform rods, AB and AC, are freely jointed at A. The weight of AB is W and its length is $6l$; the weight of AC is U and its length is λl, where λ is a constant. The rods rest in equilibrium in a vertical plane on two small smooth pegs, P and Q, which are a distance $3l$ apart at the same horizontal level. The point X is a distance l vertically below A and lies on the line joining P and Q such that $PX = 2l$ and $XQ = l$ (see diagram).

(i) Show that the magnitude of the force acting on the rod AB at P is $1.2W$, and express the magnitude of the force acting on the rod AC at Q in terms of λ and U. **[4]**

(ii) Find the value of the constant k for which $U = kW$. Find also the value of λ. **[7]**

7



A light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass m kg hangs in equilibrium attached to the lower end of the string. A small peg Q is fixed at a distance 0.75 m vertically below O. The particle P is given a horizontal velocity of $\sqrt{\frac{g}{90}}$ m s^{-1} so that the string initially moves to the left, away from the peg Q (see diagram).

(i) Prove that the motion of P while it is to the left of its initial position is approximately simple harmonic, and find the period of this motion. **[7]**

(ii) Find the time that elapses between the first and second occasions that the string makes an angle of 5° with the vertical. Find also the linear speed of P in this position. **[5]**

(iii) When the string returns to the vertical position, P begins to move in a circle with centre Q. Explain with a reason whether the motion of P is approximately simple harmonic motion while P is to the right of its initial position. **[3]**

END OF QUESTION PAPER

Answer			Marks	Guidance	
1	(i)	Impulse/momentum triangle with sides 0.8, 1.2 and 1 $\cos\theta = \dfrac{0.8^2 + 1^2 - 1.2^2}{2 \times 0.8 \times 1}$ 82.8° or 1.44 rads $1.2\sin\alpha = \sin\theta$ Angle 124°	B1 M1 A1 M1 A1 [5]	OR $1.2\cos\alpha = \cos\theta - 0.8$ $1.2\sin\alpha = \sin\theta$ M1 $1.44 = (\cos\theta - 0.8)^2 + \sin^2\theta$ A1 isw cv θ; OR from cos rule No isw	 Square and add 82.81924° or 1.445 rads may see 55.771° or 0.97339 rads 2.168 rads
2	(i)	$\frac{1}{2}m \times 0.7^2 = \frac{1}{2}mv^2 + \frac{24mg0.3^2}{2 \times 1.2} - mg \times 1.5$ Speed = 3.5 (ms^{-1})	M1 A1 A1 [3]	By energy; needs KE, PE and EE terms OR $\frac{1}{2}m \times 4.9^2 = \frac{1}{2}mv^2 + \frac{24mg0.3^2}{2\times1.2} - mg \times 0.3$ **AG** Adequate working, no errors	Allow wrong signs, missing '2'
	(ii)	One correct EE term involving *x* seen $\frac{1}{2}m \times 0.7^2 = \frac{24mg(x-1.2)^2}{2 \times 1.2} + \frac{32mg(x-1.5)^2}{2 \times 0.8} - mgx$ $[48x^2 - 136x + 95 = 0]$ 1.25 (m) and 1.58 (m)	B1 M1 A1 M1 A1 [5]	 Where *x* is distance below O OR, where *x* is dist from T, $\frac{1}{2}m \times 0.7^2 = \frac{24mg(x+0.3)^2}{2\times1.2} + \frac{32mgx^2}{2\times0.8} - mg(x+1.5)$ Leads to $48x^2 + 8x - 1 = 0$ Correct attempt to solve their 3 term quad. $1\frac{1}{4}$ $1\frac{7}{12}$	 Energy equation with at least 1 KE, 1 PE and 1 EE term and values subst. Alt left side: $\frac{1}{2}m3.5^2 + \frac{24mg0.3^2}{2\times1.2} - 1.5mg$ Dep M1 above

3	**(i)**	48 (ms^{-1})	B1 [1]	Accept ≤ 48	
	(ii)	Use $\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2a$	M1*	Accept $v\frac{dv}{dx}$ for a	Allow missing 0.2 or sign error
		$\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2\frac{dv}{dt}$	A1		
		$2.5t=\int\frac{dv}{\sqrt{12-\frac{1}{4}v}}(+c)$	*M1*	Sep variables and integrate one side	$2.5t=\int\frac{2dv}{\sqrt{48-v}}(+c)$
		$2.5t=-8\left(12-\frac{1}{4}v\right)^{\frac{1}{2}}(+c)$	A1		$2.5t=-4(48-v)^{\frac{1}{2}}(+c)$
		[c = 24]	*M1	For attempt to find c, dep previous M1	
		$v=48-4\left(3-\frac{t}{3.2}\right)^2$	A1 [6]	oe $12+7.5t-\frac{25}{64}t^2$ (0.390625)	$v=48-0.390625(9.6-t)^2$
	(iii)	$x=\int(12+\frac{24}{3.2}t-\frac{4}{3.2^2}t^2)dt$	M1	OR $x=\int(48-4\left(3-\frac{t}{3.2}\right)^2)dt$	OR $\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2v\frac{dv}{dx}$
		$x=12t+3.75t^2-0.1302t^3(+c)$	A1	$x=48t+\frac{12.8}{3}\left(3-\frac{t}{3.2}\right)^3(+c')$	via subst $\left(12-\frac{1}{4}v\right)=u^2$
		(t = 0 and) t = 3.2	M1	ft their (ii)	$x=12.8\left(12u-\frac{u^3}{3}\right)+C$
		Distance = 72.533 (m)	A1 [4]		

4	(i)	Momentum equation $2ma = -2m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} + 7m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}}$ $(a =) \frac{1}{4}$ (ms^{-1}) Comp of speed of A perp = 0.2 Speed of A was $\sqrt{(0.25^2 + 0.2^2)}$ OR $\tan\theta = \frac{0.2}{0.25}$ Speed 0.320 or $\frac{\sqrt{41}}{20}$; Ang 38.7° or 0.675 rads NLM $0.1 + 0.1 = -e(0 - a))$ $(e =)$ 0.8	M1 A1 B1 M1 A1 M1 A1 [7]	Along line of centres Allow errors with signs and masses soi soi Allow their vel comps oe For both angle and speed Along line of centres Allow errors with signs	Allow use of 63.4° for full marks Must use comp of vel 0.320156; 38.6598° or 0.67474 rads May see $\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}}$ for 0.1
	(ii)	A and B have same speed perpendicular to line of centres after first collision	B1 [1]	accept 'vertical'	
	(iii)	Momentum equation along line of centres $3mU - 2m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} = 3mc + 2ma'$ NLM $a' - c = -1(-\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} - U)$ Use $a' = 0.1$ Max $U = \frac{1}{15}$	M1 A1 M1 A1 B1 A1 [6]	Allow errors with signs and masses Allow cosα for $\frac{1}{\sqrt{5}}$ Allow errors with signs Accept any inequality Accept 0.0667 accept $\leq$	Must use comp of vel Or conservation of energy $\frac{1}{2}3mU^2 + \frac{1}{2}2m0.1^2 = \frac{1}{2}3mc^2 + \frac{1}{2}2ma'^2$ do not accept $<$
5	(i)	$3mga\cos\frac{\pi}{6}$ and $2mga\cos\frac{\pi}{6}$ $3mga\cos\left(\frac{\pi}{6}+\theta\right) + 2mga\cos\left(\frac{\pi}{6}-\theta\right) + \frac{1}{2}3mv^2 + \frac{1}{2}2mv^2$ $v^2 = \frac{2}{5}ag\left(5\cos\frac{\pi}{6} - 3\cos\left(\frac{\pi}{6}+\theta\right) - 2\cos\left(\frac{\pi}{6}-\theta\right)\right)$	B1 M1 A1 A1 [4]	Initial PE Final PE + KE **AG** Equating and correct manipulation	If O is zero level for PE For M1 at least 1 KE and 1 PE term; allow m used for 2m/3m; wrong signs; missing g

	(ii)	$v^2 = \frac{2}{5}ag\left(5\cos\frac{\pi}{6} - 3\cos\frac{\pi}{3} - 2\cos 0\right)$ $3mg\cos\frac{\pi}{3} - R = 3m\frac{v^2}{a}$	B1 M1	$v^2 = \frac{1}{5}ag(5\sqrt{3} - 7)$ $F = ma$, condone sign error; allow m used for 2m/3m	OR $3mg\cos\left(\frac{\pi}{6} + \theta\right) - R = 3m\frac{v^2}{a}$ $R = 3mg\cos\left(\frac{\pi}{6} + \theta\right) - 3m\frac{2}{5}g\left(5\cos\frac{\pi}{6} - 3\cos\left(\frac{\pi}{6} + \theta\right) - 2\cos\left(\frac{\pi}{6} - \theta\right)\right)$
		$R = 3mg\cos\frac{\pi}{3} - 3m\frac{2g}{5}\left(5\cos\frac{\pi}{6} - \frac{7}{2}\right)$	A1		$R = mg(6.6\cos\left(\frac{\pi}{6} + \theta\right) + 2.4\cos\left(\frac{\pi}{6} - \theta\right) - 6\cos\frac{\pi}{6})$
		$R = mg\left(5.7 - 6\cos\frac{\pi}{6}\right)$ oe	A1 [4]	Accept 0.5038475mg or $mg(5.7 - 3\sqrt{3})$ oe	Answer must be simplified 4.94m loses last mark
6	(i)	$Pl\sqrt{5} = W \times 3l\cos\theta$ $P = 1.2W$ $Ql\sqrt{2} = U \times \frac{\lambda}{2}l\cos\emptyset$ $Q = 0.25\lambda U$	M1 A1 M1 A1 [4]	Mom about A for AB **AG** Mom about A for AC	Allow $\sin\theta$, cancelled l Not from use of angle 26.565° Allow $\sin\emptyset$, cancelled l
	(ii)	(H) $P\sin\theta = Q\sin\emptyset$ (V) $W + U = P\cos\theta + Q\cos\emptyset$ $W + U = P\cos\theta + P\sin\theta \times \frac{\cos\emptyset}{\sin\emptyset}$ $W + U = \frac{3}{\sqrt{5}} \times 1.2W$ $k = 0.610$ $\lambda = 4.98$ $[P\sqrt{5}l - W3l\cos\theta = Q\sqrt{2}l - U\frac{\lambda}{2}\cos\emptyset$	M1 M1 A1 M1* *M1 A1 A1 [7]	$P\frac{1}{\sqrt{5}} = Q\frac{1}{\sqrt{2}}$; compts essential $W + U = P\frac{2}{\sqrt{5}} + Q\frac{1}{\sqrt{2}}$; compts essential Eliminate Q (or P) dep M1M1 Elim P and Q to get equation in k, $W + U =$ 1.609689W $\frac{18\sqrt{5} - 25}{25}$ Mom about A (or any other point) for whole system – allow M1(A1) if resolving not seen twice]	Allow $\frac{2}{\sqrt{5}}$ for M1 Allow $\frac{1}{\sqrt{5}}$ for M1, sign errors $W + U = Q\cos\theta \times \frac{\sin\emptyset}{\sin\theta} + Q\cos\emptyset$ $\left[W + U = 0.25\lambda U \times \frac{3}{2\sqrt{2}}\right]$ 0.6099689 4.97695 Allow use of angles in (ii): 26.6 & 45 OR after M1M1A0/1, M1* for 2 equns in terms of k and λ, *M1 for solving for k or λ.

7	**(i)**	$\frac{1}{2}m \times \frac{g}{90} = mgh$ [Max height = $\frac{1}{180}$ = 0.005556] Max angle = 6.76° or 0.118 rads $-mg\sin\theta = m \times 0.8 \times \ddot{\theta}$ $\ddot{\theta} = -\frac{9.8}{0.8}\theta$, SHM (about θ = 0) since θ is small $\omega^2 = 12.25$ Period = 1.80 secs ($\frac{4}{7}\pi$)	M1 A1 M1 A1 A1 M1 A1 [7]	By energy; allow cancelled m Allow M1A1 for 6.76° or 0.118 rads in (ii) N2L; allow a for 0.8 $\ddot{\theta}$; allow cancelled m Cand value	$\frac{1}{2}m \times \frac{g}{90} = mg \times 0.8(1-\cos\theta)$ 6.756 / 0.11798 allow sign error, sin / cos 1.7952
	(ii)	$0.087266 = A\sin 3.5t$ t = 0.238 secs $t' = 2\left(\frac{1.7952}{4} - 0.2378\right)$ = 0.422(s) $\dot{\theta} = 0.118 \times 3.5\cos 3.5 \times 0.238$ Linear speed = 0.222(ms^{-1})	M1 A1 A1 M1 A1 [5]	OR $5 = A\sin 3.5t$; A = amplitude Or 0.65972 Or 0.65972 – 0.2378 OR $\dot{\theta} = \sqrt{(3.5^2(0.118^2 - 0.0873^2)}$ 0.8 ×0.278	May use cos * 0.2378 allow sin if consistent with *; allow 5° and 6.76° Or $\frac{1}{2}m\frac{9}{90} = \frac{1}{2}mv^2 + mg0.8(1 - \cos 5°)$
	(iii)	Max height is still 0.00556 soi Max angle = $\cos^{-1}\frac{(0.05-0.00556)}{0.05}$ [27.3] Not SHM since angle is not small	B1 B1 B1 [3]	accept 'still the same'	or attempt to work out height 0.476 rads